Numerical Models for Inertial Electrostatic Confinement Fusion with a High Ion Injection Current

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Abstract

An inertial electrostatic confinement (IEC) fusion device traps plasma particles with a spherically symmetric electrostatic field that accelerates ions toward the device's center, where they can collide and potentially fuse. The IEC fusion device is simpler and less expensive to construct when compared to magnetic confinement devices, like the tokamak: its employment will benefit fusion research. Here, the plasma motion in an IEC device is numerically simulated by solving the Vlasov-Poisson system of partial differential equations in spherical coordinates with one dimension of space and one dimension of momentum. The stability and accuracy of solutions obtained by various numerical methods, including the finite element method and the finite volume method, are compared.

Vlasov-Poisson Equations

The Vlasov-Poisson equations describe the time evolution of a collisionless plasma under the effects of an electrostatic field (electrostatic limit of the Vlasov-Maxwell equations):

$$\text{Vlasov:} \quad \frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \frac{\partial f}{\partial \vec{r}} + q\vec{E} \cdot \frac{\partial f}{\partial \vec{p}} = 0$$

$$\text{Poisson:} \quad \nabla \cdot \vec{E} = -\frac{\rho}{\epsilon_0}$$

where independent variables t, $\vec{r}$, $\vec{p}$ are time, position, and momentum; $f(t, \vec{r}, \vec{p})$ is the phase-space density of plasma particles with mass m and charge q; $\vec{E}(\vec{r})$ is the electric field; and $\rho(\vec{r})$ is the spatial charge density.

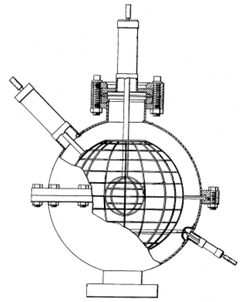
Homogeneity of the Vlasov equation corresponds to the fact that the plasma is collisionless. Otherwise, we would replace the RHS with a collision term, as in the Fokker-Planck equation.

We are interested in a spherically symmetric system, where the Vlasov-Poisson equations become one-dimensional (the radial dimension):

$$\text{Vlasov:} \quad \frac{\partial f}{\partial t} + \frac{p}{m}\frac{\partial f}{\partial r} + qE\frac{\partial f}{\partial p} = 0$$

$$\text{Poisson:} \quad \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 E\right) = -\frac{\rho}{\epsilon_0}$$

Modeling the IEC Device



Hirsch-Meek fusor

Solution mesh: Solve for the values of f and E over the 2D grid: radius $\times$ radial momentum, and step forward in time.

Boundary conditions:

- $E = 0$ on all four boundaries.
- $f = 0$ on the max and min momentum; choose p_{max} large enough that approximately no particles approach it.
- Periodic boundary: $f(r = 0, p) = f(r = 0, -p)$; at the center of the device, incoming ($p < 0$) particles become outgoing particles ($p > 0$) with the same magnitude of momentum.
- Ion source: ions are injected into the system at r_{max} at rest with a current of I_0. At the r_{max} boundary: $\mathcal{I} \equiv df = (I_0\, dt)/(q\, dr\, dp)$ is added to a discrete number of cells near $p = 0$ at each time step.

We present numerical solutions to the Vlasov-Poisson equations (up to normalization of units) with these boundary conditions.

Finite Volume Method (FVM)

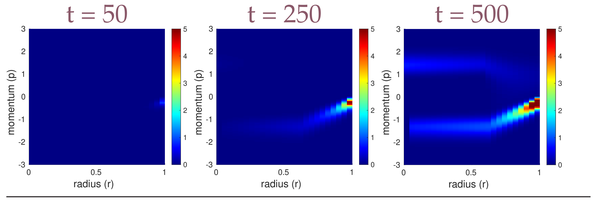
In the FVM, we compute the flux terms $G^k_{i,j}$ through the boundaries of each sub-cell in the domain to determine the time evolution of f. For square sub-cells, each with four boundaries, the discretized conservation form of the Vlasov equation is solved as:

$$f^{k+1}_{i,j} = f^k_{i,j} - \frac{\Delta t}{\Delta r \Delta p} \sum_{\beta=1}^{4} G^k_{i,j;\beta}$$

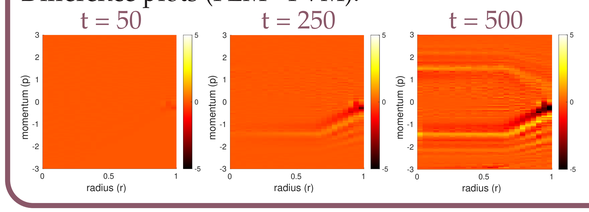
Electric field at each position r_i is solved as:

$$E(r_i) = \frac{q(r_i) + Q}{4\pi\epsilon_0 \cdot r_i^2}$$

where q is enclosed plasma charge, Q is effective charge due to cathode. The solutions f, E are computed in MATLAB, and the ion phase-space density f is shown below for three time values:



Difference plots (FEM - FVM):



Finite Element Method (FEM)

In the FEM, we solve the weak form of the Vlasov equation for f using the FreeFEM software:

$$\int_\Omega \left(\left[\frac{f - f_{\text{old}}}{\Delta t} \right] + \frac{p}{m}\frac{\partial f}{\partial r} + qE\frac{\partial f}{\partial p} \right) v \, d\Omega = 0$$

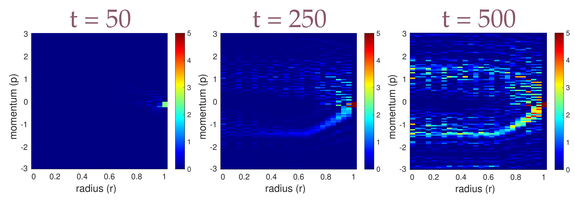
We solve for the electric field in two stages. First, the electric field due to plasma charges E_α is found by solving the weak form of the Poisson equation:

$$\int_\Omega \left(\left[\frac{2}{r}E_\alpha + \frac{\partial E_\alpha}{\partial r} \right] + \frac{\rho}{\epsilon_0} \right) w \, d\Omega = 0$$

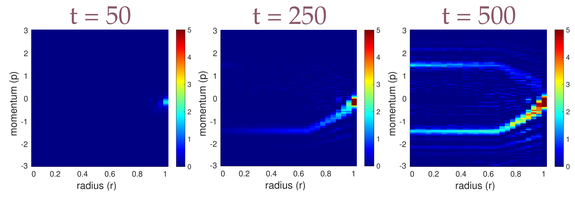
Then, the total electric field including effects of the cathode voltage V_C at position r_C is:

$$E(r) = E_\alpha(r) + \frac{V_C + \int_{r_C}^{r_{\text{max}}} E_\alpha(r) dr}{r^2(1/r_C - 1/r_{\text{max}})}$$

Before Homogenizing



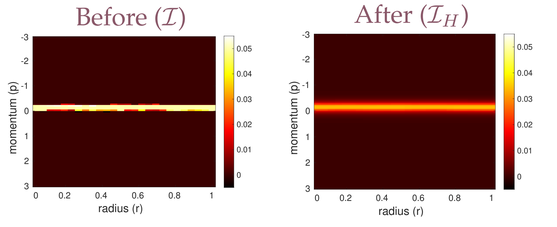
After Homogenizing



To reduce noise, we homogenize the ion injection source term $\mathcal{I} = (I_0\, dt)/(q\, dr\, dp)$. The homogenized version $\mathcal{I}_H$ is obtained by solving:

$$\mathcal{I}_H - \ell^2 \Delta \mathcal{I}_H = \mathcal{I}, \qquad \left.\frac{\partial \mathcal{I}_H}{\partial n}\right|_{\partial\Omega} = 0$$

with parameter $\ell = 0.1$.



Future Plans:

- Continue optimizing FEM solution to reduce noise.
- Investigate using spectral (Fourier) methods for computing the Vlasov-Poisson solution.
- Normalize units to compare simulation with experimental data.

References

[1] **J. Black, M. Wood-Thanan, A. Maroni, and E. Sánchez.** *Study of inertial electrostatic confinement fusion using a finite-volume scheme for the one-dimensional Vlasov equation*. Phys. Rev. E, **103**(2), 023212, 2021.

[2] **G. H. Miley, S. K. Murali.** Inertial Electrostatic Confinement (IEC) Fusion. Springer, 2014.