Testing the Relativistic-Microwave Theory of Ball Lightning with Plasma Simulation

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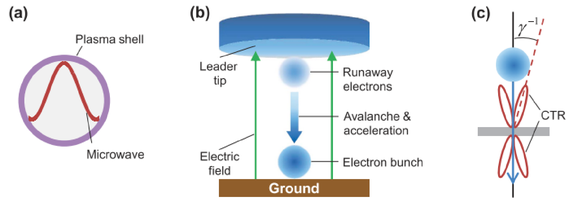
Introduction

Ball lightning (**BL**) is an unexplained phenomenon reported by thousands of eyewitnesses as a fire-ball, a few cm to 1 m in diameter, moving unpredictably and independently of the wind, sometimes observed during lightning storms. Here a potential theory for the creation of BL is explored [1].



1. In the last leader step of a lightning strike, a bunch of runaway electrons emerges from the leader tip, is accelerated to a relativistic speed by the electric field between the leader and ground, and undergoes an avalanche.
2. Coherent transition radiation (**CTR**) is produced by the electron bunch striking the ground or passing through aircraft skins.
3. This intense wave is trapped in a plasma shell.

Mathematical Model

Normalization: to laser-related units

$$\frac{\mathbf{r}}{\lambda}, \frac{2\pi t}{\omega}, \frac{e\mathbf{E}}{m_e\omega c}, \frac{e\mathbf{B}}{m_e\omega}, \frac{\rho}{en_c}, \frac{\mathbf{J}}{en_c c}, \frac{\mathbf{P}}{Mc}, \frac{M}{m_e}, \frac{\mathbf{V}}{c}, \frac{q}{e},$$

with reference variables defined as:

- λ: the laser wavelength in vacuum
- ω: the laser angular frequency
- n_c: critical plasma density
- m_e: electron mass
- M: can be the electron or ion mass

Unitless Maxwell-Lorentz equations:

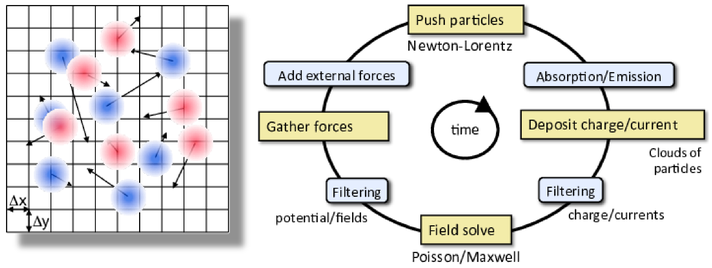
$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$$
$$\nabla \times \mathbf{B} = \partial_t \mathbf{E} + 2\pi\mathbf{J}$$
$$\nabla \cdot \mathbf{E} = 2\pi\rho$$
$$\frac{d\mathbf{P}}{dt} = \frac{2\pi q}{M}(\mathbf{E} + \mathbf{V} \times \mathbf{B})$$

Computational Methods

An open-source Particle-in-Cell (**PIC**) code called **Smilei** [2] is used to simulate two parts of the theory:

1. Microwave pulse generation due to **CTR**
2. Microwave bubble trapping

PIC Code Algorithm Loop [3]



Vlasov-Maxwell Collisionless Plasma
Each particle species of the plasma is described by a distribution function, $f_s(t, \mathbf{x}, \mathbf{p})$, satisfying

$$\left(\partial_t + \frac{\mathbf{p}}{m_s\gamma} \cdot \nabla + \mathbf{F}_L \cdot \nabla_{\mathbf{p}}\right) f_s = 0,$$

with the relativistic Lorentz factor γ and the Lorentz force $\mathbf{F}_L$:

$$\gamma = \sqrt{1 + \mathbf{p}^2/m_s^2}$$
$$\mathbf{F}_L = q_s(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

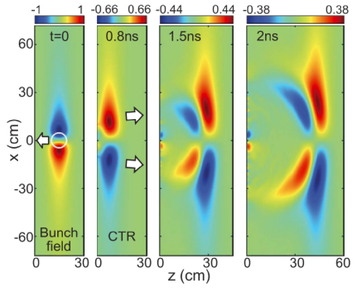
Relativistic Equations of Motion
The particle's momentum $\mathbf{p}$ and position $\mathbf{x}$ are computed using a second order leap-frog integrator derived from:

$$\frac{d\mathbf{x}_p}{dt} = \frac{\mathbf{u}_p}{\gamma}$$
$$\frac{d\mathbf{u}_p}{dt} = r_s\left(\mathbf{E}_p + \frac{\mathbf{u}_p}{\gamma_p} \times \mathbf{B}_p\right),$$

where $r_s = q_s/m_s$ is the charge-over-mass ratio for species s, and $\mathbf{u}_p = \mathbf{p}_p/m_s$ is the reduced momentum.

Desired Results

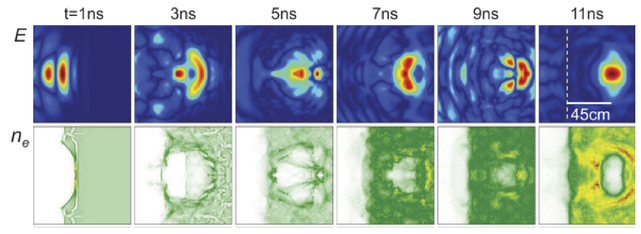
Simulation 1 – Desired Result [1]
Electric field with peak amplitude ≈ 310 MV/m generated by relativistic electrons accelerated towards conductor. Simulated with open boundaries.



- Conductor at $z = 0$ as over-dense, cold, neutral plasma with density $n_0 e^{-z^2/2\sigma^2}$
  - $n_0 = 50n_c$
  - $n_c = \epsilon_0 m\omega^2/e^2$
  - $\sigma = 4$cm
- Electron bunch with 0.0001 eV temperature, 0.01c speed, and density $n_{b0} = 3.7 \cdot 10^{11}\ cm^{-3}$ and $n_{b0}exp[-(x^2 + z^2)/(2\sigma^2)]$

Simulation 2 – Desired Result [1]
Microwave-bubble trapping previously demonstrated.

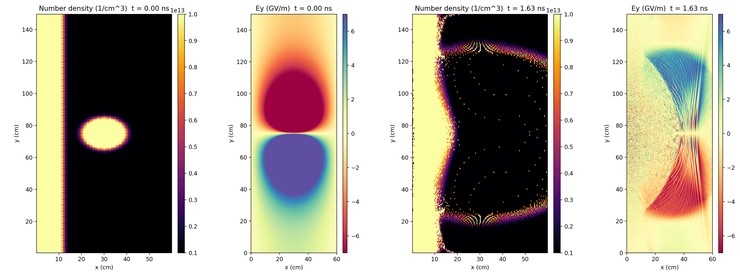
Field input:
$E_z = E_0 exp(-y^2/R^2) sin^2[\pi(t - x/c)/\tau] sin[\omega(t - x/c)]$



- E_0 = 310 MV/m
- R = 9cm
- $\tau \leq 2ns$
- $\omega/2\pi$ = 1 GHz
- λ = 30 cm
- Conductor: $n_0 = 4n_c$.

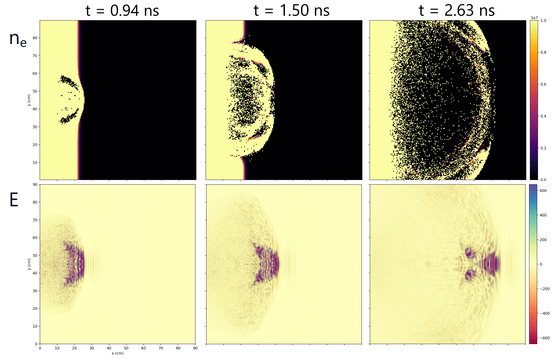
Results

Simulation 1 – Results
Maximum electric field magnitude $|E_y| \approx 7$ GV/m
Number density of electrons $\approx 10^{14}$



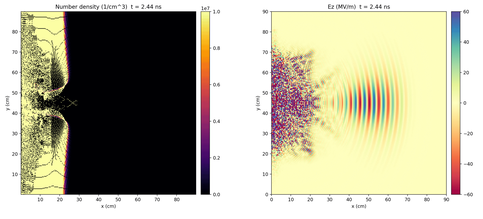
Electron bunch structure is not stable during acceleration.

Simulation 2 – Results: Plasma shell formation is resolution dependent. Ions are not mobile



256 grids/λ, $\tau = 0.3ns$

← Unstable spherical shell formation.
↓ Coherent wave packet forms with desired width of 30-45 cm.



80 grids/λ in x, 64 grids/λ in y, $\tau = 2ns$

References

[1] H-C. Wu. *Relativistic-Microwave Theory of Ball Lightning*. Scientific Reports, **6**, 28263, 2016. • [2] SMILEI 4.7. *https://smileipic.github.io/Smilei/index.html* • [3] Particle-in-Cell Method. WarpX 23.02. *https://warpx.readthedocs.io/en/latest/theory/picsar_theory.html*